Endogenous Uncertainty and Investment Commitment*

Lars-Alexander Kuehn
Tepper School of Business
Carnegie Mellon University

January 10, 2019

Abstract

An important feature of aggregate consumption data is the presence of time-varying volatility. This paper shows how these volatility dynamics can arise *endogenously* in a DSGE model with Epstein-Zin preferences and investment commitment. Investment commitment captures the idea that long-term investment projects require not only current expenditures but also commitment to future expenditures. Even though the model is driven by random walk productivity, the general equilibrium effects of investment commitment and Epstein-Zin preferences generate endogenously countercyclical consumption growth and stock return volatility as in the data.

JEL Classification: E32, G12

Keywords: Macroeconomic uncertainty, GARCH, long-term capital expenditures, Epstein-Zin utility.

*This article is a based on my PhD thesis at the University of British Columbia. I am grateful to my thesis committee Murray Carlson, Adlai Fisher, Henry Siu and Tan Wang for constant support and encouragement. I also thank Harjoat Bhamra, Oliver Boguth, Ilan Cooper, Urban Jermann, Ron Giammarino, Marcin Kacperczyk, Michael Mueller, Hongjun Yan, Lu Zhang, seminar participants at UBC, University of Arizona, University of Texas at Austin, Boston College, Carnegie Mellon University, Columbia University, University of Texas at Dallas, Duke University, LBS, LSE, Imperial College, NYU, Ohio State University, University of Illinois at Urbana-Champaign, University of Toronto, Vanderbilt University, and conference participants at the Society for Economic Dynamics Conference, the European Finance Association, and the American Finance Association meetings for valuable comments. Contact information: 5000 Forbes Avenue, Pittsburgh, PA 15213, `kuehn@cmu.edu`.

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1 Introduction

Macroeconomic uncertainty varies with the business cycle. Specifically, consumption and output growth volatilities are strongly countercyclical in post-war US data. I quantify this asymmetric response of volatilities to positive versus negative aggregate shocks with the EGARCH specification of Nelson (1991). Time varying uncertainty also causes the unconditional distribution of consumption and output growth rates to deviate from normality. In particular, both consumption and output growth are negatively skewed and have positive excess kurtosis.

Motivated by these empirical regularities, there now exists a growing literature that emphasizes exogenous volatility shocks as the driver of business cycles and firm policies (e.g., Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano and Primiceri (2008), Christiano, Motto, and Rostagno (2009), Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)). In contrast, the contribution of this paper is to show that countercyclical volatility and output dynamics can arise *endogenously* in DSGE models. To this end, I depart from the literature along two dimensions. First, I drop the assumption of instantaneous investment and model the commitment of long-term investment projects. Second, I embed this investment friction in a general equilibrium model where the representative agent has Epstein-Zin preferences.[1]

The key building block of my model is a new and tractable specification of long-term investment projects. A common assumption is that investment occurs instantaneously. This assumption is obviously unrealistic. For example, expanding capacity in a manufacturing process can take several years. A more realistic assumption is that an investment decision results in a series of both expenditures and capital increases over time. These long-term investment projects are in addition costly to undo because of contractual agreements between sellers and suppliers. Consequently, investments in long-term projects imply a commitment on the part of firms. Investment commitment is distinct from traditional time-to-build, e.g. Kydland and Prescott (1982). The time-to-build literature focuses on the lag between the investment decision and the time when the new project becomes productive. My focus is instead on the equilibrium consequences of investment projects which involve not only current expenditures but also commitment to future expenditures.

When investment projects take place over time, the distribution of initiated projects be-

[1] See also Fernandez-Villaverde and Rubio-Ramirez (2010).

comes a high dimensional state variable and renders the problem intractable. The approach I take solves this problem by modeling investment projects as perpetual, with costs declining geometrically over time. This formulation allows me to summarize the total costs of prior projects with a single additional state variable—lagged investment expenditures. In addition, I assume that firms cannot undo previous investment decisions. Initiated projects therefore represent commitment, and I call the resulting capital friction *investment commitment*.

This investment commitment friction generates interesting dynamics in consumption and prices. After a positive technology shock, firms initiate new investment projects to take advantage of higher productivity. As a result, the commitment level increases. After an adverse shock, the household would like to lower investment to smooth consumption over time. However, prior commitments oblige the firm to complete initiated projects, so that consumption is constrained by the history of investment decisions, making a negative shock worse after a boom than a bust. Moreover, commitments in long-term projects are not satisfied immediately, thereby depressing consumption for several consecutive periods. Thus, a negative shock adversely affects consumption in the current as well as future periods.

These investment commitments matter qualitatively when the agent has Epstein-Zin preferences. The advantage of Epstein-Zin preferences is that they achieve a separation of relative risk aversion and elasticity of intertemporal substitution. With power utility, a realistic level of risk aversion implies a low elasticity of intertemporal substitution because the two concepts are inversely related. To generate a realistic movement in prices, the household has to be fairly risk averse. At the same time, for investment commitment to have a significant impact on quantities, the household needs to have a high elasticity of intertemporal substitution. Both are not possible with power utility. Only when the agent is willing to delay consumption over time (high elasticity of intertemporal substitution) does she invest a larger fraction of her wealth in real assets. With such a parameterization, the commitment level rises dramatically in good times leading to a frequently binding commitment constraint when the economy switches into a recession. Conversely, with a low elasticity of intertemporal substitution, the commitment constraint has little impact on consumption and prices.

To demonstrate that investment commitment has not only a momentary but a long lasting impact on consumption, I estimate an EGARCH model on simulated data. As in US data, the volatility of consumption and output growth and stock returns are time varying and strongly countercyclical. The so-called leverage parameter, which measures the asymmetric response of volatility to positive versus negative shocks, is negative as in the data, meaning that negative shocks raise volatility. In addition, consumption, output growth and equity returns are negatively skewed and have positive excess kurtosis. To further illustrate that the volatility dynamics are tight to investment and price dynamics, I show that the aggregate market-to-book ratio, i.e., Tobin's Q, positively predicts consumption consumption and output growth as well as equity return volatilities.

The countercyclicality and predictability of volatilities arises in the model since investment commitment, similar to irreversible investment,[2] is an asymmetric friction. It prevents firms from disinvesting but does not hinder firms from investing. This feature implies that the impact of the friction varies over the business cycle.

While research has made significant progress towards understanding the effects of irreversible investment at the firm level, very little is known about the aggregate consequences of non-convex investment frictions.[3] This paper demonstrates that a non-convex friction can be binding even in an aggregate model and can thereby impact aggregate quantities and prices. The constraint of the commitment friction binds because the lower bound of investment is not zero, as in the standard irreversible investment friction, but history dependent and a function of the firm's prior investment decisions. Essentially, commitments bound the growth rate of investment in bad times. As such, investment commitment can be understood as micro foundation for adjustment costs in investment growth rates as suggested by Christiano, Eichenbaum, and Evans (2005).

Empirical evidence supporting investment commitment can be found in the literature. Lamont (2000) and Jones and Tuzel (2012) find that investment plans or orders, instead of actual investment, forecast stock returns because these orders represent future commitments. Eberly, Rebelo, and Vincent (2012) find that the best predictor of current investment at the firm level is lagged investment. In my model, lagged investment measures investment commitment.

[2]See, for instance, Barnett and Sakellaris (1998) and Abel and Eberly (2002), who report empirical evidence of non-convex capital friction.

[3]The aggregate consequences of non-convex capital frictions is an ongoing debate in the literature. Early papers, for instance Veracierto (2002), Thomas (2002), Khan and Thomas (2008), reach the conclusion that mirco lumpiness does not matter for aggregate quantities. More recently, Bachmann, Caballero, and Engel (2012) reach the opposite conclusion.

My paper is also closely related to the business cycle literature studying the general equilibrium asset pricing effects of investment frictions. In line with the equity premium puzzle in an endowment economy, Rouwenhorst (1995) demonstrates the failure of the standard business cycle model to account for the equity premium. Jermann (1998) and, more recently, Boldrin, Christiano, and Fisher (2001) show that business cycle models can generate a reasonable equity premium when they are enhanced with investment frictions and internal habit preferences. Jermann (1998) assumes convex capital adjustment costs and Boldrin, Christiano, and Fisher (2001) inter-sector capital and labor immobility. Kogan (2001, 2004) analyzes the effects of irreversible investment on stock returns in a two-sector general equilibrium model.[4]

Commitment has been of interest in other strands of the literature, too. Chetty and Szeidl (2010) model consumption commitment directly as a friction of the consumption process, thereby providing a foundation for habit formation. Eisfeldt and Rampini (2009) consider the effect of committed dividend payments on corporate liquidity demand. Carlson, Fisher, and Giammarino (2010) show how investment commitment affects risk dynamics in a partial equilibrium real options framework.

The paper has the following structure: In Sections 3 and 4, I explain the optimization problem of the firm and household, respectively. Section 5 contains the numerical results of the model. Section 6 concludes.

2 Empirical Evidence

In this section, I summarize the empirical evidence on the higher moments of consumption, output and stock return data. Consumption growth is measured as the log growth rate of the sum of real nondurable and service consumption consumption expenditures or as real total personal consumption expenditures. Output growth is the log growth rate of the real gross domestic product. Aggregate stock returns are the value-weighted CRSP index. All data is quarterly and covers the period form 1952 to 2011.

I quantify the time varying nature of volatilities using the EGARCH framework of Nelson (1991). The EGARCH model relative to the standard GARCH model has the advantage that

[4] More recently, Kaltenbrunner and Lochstoer (2010), Campanale, Castro, and Clementi (2010), and Croce (2012) embed Epstein-Zin preferences in a production economy and analyze its impact on stock returns. These papers rely on convex adjustment costs and therefore cannot generate countercyclical predictable moments. Gourio (2012) studies disaster risk in business cycle models. Guvenen (2009) studies asset prices in a two-agent macroeconomic model with limited stock market participation and preference heterogeneity.

it explicitly measures the impact positive versus negative shocks on volatility dynamics. Say, consumption growth follows an EGARCH(1,1) specification, then $\Delta c_t = g_t + \sigma_t \epsilon_t$ where g_t captures the potentially time varying mean of consumption growth, σ_t denotes the volatility of consumption growth, and ϵ_t are standard normal innovations, $\epsilon_t \sim \mathcal{N}(0,1)$. The EGARCH model postulates that the log variance follows[5]

$$\ln \sigma_t^2 = \kappa + \rho \ln \sigma_{t-1}^2 + \lambda(|z_{t-1}| - \mathbb{E}|z_{t-1}|) + \theta z_{t-1} \qquad z_t = \epsilon_t / \sigma_t \tag{1}$$

where κ is the average log variance, ρ measures the persistence of the log variance, λ the size of the impact of shocks on volatility and θ its sign. Importantly, when increases in volatility are driven by adverse shocks, θ is negative, capturing the so-called leverage effect.

Table 1 summarizes the parameter estimates for an EGARCH(1,1) model for consumption, output and stock return data. For all three time series, volatility is highly persistence (large ρ) and very volatile (large λ). Importantly, increases in volatility are driven by adverse shocks ($\theta < 0$), meaning volatility is countercyclical. To illustrate the counter cyclicality of volatilities, Figures 1-4 display the annualized EGARCH volatility estimate for real nondurable plus service consumption, real personal consumption expenditure, output and aggregate stock returns, respectively. Grey bars represent NBER recessions.

Time varying uncertainty also causes the unconditional distribution of consumption and output growth rates to deviate from the normal distribution. Table 1 also reports unconditional skewness and kurtosis. In particular, both consumption and output growth as well as stock returns are negatively skewed and have positive excess kurtosis. Negative skewness supports the claim that volatility dynamics are driven by bad news. Positive excess kurtosis could be driven by time varying rare events or uncertainty. Since the data starts in 1952 the rare event hypothesis is an unlikely explanation and the time varying uncertainty the more plausible one. The goal of the model is to explain not only the volatility dynamics but also the higher moments of consumption, output and stock return data.

3 Firm

The model economy consists of two agents: a representative household and firm. The firm owns the capital stock of the economy and chooses optimal real investment. The household

[5] With standard normal innovations, it follows that $\mathbb{E}z = \sqrt{2/\pi}$.

trades in the stock and bond market and receives dividend income from holding the firm's stocks. In equilibrium, the household has to hold all stocks and the risk-free bond is in zero net supply.

The firm's objective is to maximize firm value P_t by making optimal real investment decisions I_t:

$$P_t = \max_{\{I_{t+s}\}_{s=1}^{\infty}} \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t+s} D_{t+s} \right] \qquad D_t = Y_t - I_t$$

where M_t denotes the stochastic discount factor and D_t the dividend payment to the shareholder. Dividends are defined as the residual payment after subtracting investment I_t from output Y_t. Output is determined by a Cobb-Douglas production function F

$$Y_t = Z_t^{1-\alpha_1} K_t^{\alpha_2}$$

where K_t denotes capital, Z_t an exogenous technology shock, and α_2 the capital share of production.

The technology shock follows a difference stationary process

$$\ln\left(\frac{Z_{t+1}}{Z_t}\right) = \Delta z_{t+1} = (1-\varrho)g + \varrho \Delta z_t + \sigma \varepsilon_{t+1}$$

where g is the growth rate of the economy, ϱ is the auto-correlation, and ε_t is an i.i.d. standard normal innovation. In many production economies the technology shock follows an AR(1) process. The resulting dynamics of the model are then mainly driven by the exogenous shock process. In contrast, all dynamics in this model arise endogenously from the investment friction because the shock process follows a random walk.

The crucial ingredient of a production economy is the investment friction. In response to demand shocks to the economy, prices and the supply of capital change. Depending on the elasticity of capital, prices react more or less strongly. In the extreme case of an exchange economy, allocations are fixed (i.e., fully inelastic) and shocks are absorbed by price variation. In a production economy without an investment friction, the supply of capital is fully elastic and prices are as volatile as real capital. Investment frictions reduce the elasticity of capital and therefore cause prices to be more volatile than capital, which is necessary to generate interesting asset pricing dynamics.

3.1 Non-Instantaneous Investment

In this paper, I entertain the realistic friction that investment projects are not completed instantaneously. Before going into the details of the investment friction studied in this paper, I first provide a general framework for non-instantaneous investment. Consider a firm that decides about a new investment project of size X_t in period t. In the most general form, non-instantaneous investment has two modeling implications: First, investment expenditures in period t are a function of all previous investment project choices:

$$I_t = f_t(X_t, X_{t-1}, ..., X_0) \tag{2}$$

Second, next period's productive capital stock is a function of current capital and initiated projects:

$$K_{t+1} = (1 - \delta)K_t + g_t(X_t, X_{t-1}, ..., X_0) \tag{3}$$

The functional form of f determines when the firm has to pay for a new project and g determines when a new project adds to the productive capital stock.

The instantaneous investment model is a special case, in which the firm incurs the total costs of the new project in the current period, i.e. $I_t = X_t$, and the new project becomes productive in the next period, i.e. $K_{t+1} = (1 - \delta)K_t + X_t$.

The first paper to consider non-instantaneous investment in a production economy is Kydland and Prescott (1982). They label their investment friction time-to-build. It has two distinct features: First, it takes four quarters for an investment project to be finished and the costs are spread over this period according to the weights $w_1, ..., w_4$. This implies that

$$I_t = f(X_t, X_{t-1}, X_{t-2}, X_{t-3}) = w_1 X_t + w_2 X_{t-1} + w_3 X_{t-2} + w_4 X_{t-3}$$

where $w_1 + ... + w_4 = 1$.

Second, a new investment project increases the productive capital stock only after the investment project is completed and the total costs are incurred:

$$K_{t+1} = (1 - \delta)K_t + g(X_{t-3}) = (1 - \delta)K_t + X_{t-3}$$

The idea of the time-to-build friction is to capture a gestation lag between the investment decision and when it becomes productive. Importantly, there is no restriction placed on the investment decision X_t—it can be *negative* so that the firm can achieve any desired investment expenditure level I_t in a given period. The gestation lag shows up in the law of motion of capital. The time-to-build friction has the drawback of too many state variables. Specifically, with four periods time-to-build, the model has five state variables, namely, K_t, K_{t+1}, K_{t+2}, K_{t+3} and the technology shock—the curse of dimensionality shows up.[6]

3.2 Investment Commitment

To have a tractable specification of investment commitment, I make two assumptions: First, I place more restrictions on the dynamics for investment expenditures (2). Second, I drop the gestation lag idea of time-to-build and assume that partially completed projects add to the capital stock, i.e. $f = g$ in Equations (2) and (3).

My first assumption is that investment projects are perpetual, with costs declining geometrically over time. To gain a better understanding of this assumption, consider the case where the firm initiates a project of size X_0 at time 0 and nothing thereafter, i.e. $X_1 = X_2 = ... = 0$. In each period, the firm pays a fraction of the total project costs according to the weights $(\frac{1-w}{w})w$, $(\frac{1-w}{w})w^2$, $(\frac{1-w}{w})w^3$, etc. Thus, investment expenditures are given by

$$I_0 = \left(\frac{1-w}{w}\right) wX_0 \quad I_1 = \left(\frac{1-w}{w}\right) w^2 X_0 \quad I_2 = \left(\frac{1-w}{w}\right) w^3 X_0 \quad \ldots$$

As a consequence, the firm incurs the total costs of X_0 over the infinite future, i.e. $I_0 + I_1 + I_2 + ... = X_0$. This follows since the weights, normalized by $(\frac{1-w}{w})$, add up to 1.[7]

In general, the firm initiates new projects every period and the investment expenditures at time t are a weighted sum of all ongoing projects with respective costs X_{t-s}:

$$I_t = \frac{1-w}{w} \left(wX_t + w^2 X_{t-1} + ... + w^{t+1} X_0\right) \tag{4}$$

For instance, suppose the firm initiates a project at time 0 and 1. The investment expenditures at time 1, I_1, then consists of two terms: $I_1 = (\frac{1-w}{w})(wX_1 + w^2 X_0)$. The first term, wX_1, relates to the just initiated period 1 project. It requires current expenditures of $(1-w)X_1$. The second term captures current costs of last period's projects X_0, amounting to $(1-w)wX_0$. In Equation (4), $w \in (0,1)$ determines the degree of commitment. High w means that few of the investment costs are incurred in the current period and therefore commitment is high.

[6] Kydland and Prescott (1982) solve the model with a linear-quadratic approximation and Christiano and Todd (1996) use a log-linear approximation. Both methods rely on the certainty equivalence and therefore are not applicable to study price dynamics. Another drawback of the model are jigsaw-like impulse response functions which do not resemble empirical estimates; see e.g. Christiano and Todd (1996).

[7] Note that the weights are a geometric series, i.e., $\sum_{s=1}^{\infty} w^s = \frac{w}{1-w}$ if $|w| < 1$.

The capital expenditure equation (4) suggests that the solution is history dependent. However, at time t, I_{t-1} captures all information about previous investment decisions because the weights in Equation (4) decay geometrically over time—in contrast, in the standard time-to-build formulation the weights are free parameters. The geometric weighting implies that the investment expenditures follow a recursive law of motion

$$\begin{aligned} I_t &= (1-w)X_t + \frac{1-w}{w}\left(w^2 X_{t-1} + w^3 X_{t-2} + ... + w^{t+1} X_0\right) \\ &= (1-w)X_t + wI_{t-1} \end{aligned} \tag{5}$$

Consequently, the current capital stock K_t and last periods investment expenditures I_{t-1} are the only endogenous state variables. The reduction in the number of endogenous state variables significantly reduces the computational complexity of the model.

Without any constraint on X_t, the investment expenditure law (5) would not impose any restriction on the investment behavior of the firm. To ensure that current investment decision are irreversible in the future and therefore represent commitment, I impose the constraint that firms can only initiate projects of non-negative size

$$X_t \geq 0 \tag{6}$$

Substituting (6) into (5) gives a lower bound on investment expenditures

$$I_t \geq wI_{t-1} \tag{7}$$

which I will call the *commitment constraint*.[8] It captures the notion that firms cannot reduce investment expenditures more quickly than at the rate at which they complete unfinished projects. The lower bound on investment expenditures (7) is similar to the irreversibility constraint, $I_t \geq 0$. However, there are two major differences:

First, the lower bound in this model depends on the decisions of the firms. When firms decide to invest more after a good technology shock, the investment bound increases and reduces the flexibility of the firm in bad times. The bound is therefore an endogenous outcome of the model and *state-dependent*. Second, it is well-known that the irreversibility constraint is never binding in aggregate models because optimal investment is never negative due to

[8]The recursive law for investment looks similar to a habit model in consumption. The difference is that habit implies a lower bound on consumption because consumption has to remain above the habit level. In contrast, commitment constraint (7) implies an upper bound on consumption because of the aggregate resource constraints.

the small standard deviation of the aggregate technology shock. The commitment constraint, however, is binding in this model when w is large enough.

Effectively, the commitment constraint (7) is a lower bound on the growth rate of investment: $I_t/I_{t-1} \geq w$. This means intuitively that management can cut investment expenditures by alone $1-w$ in a given period. As such, investment commitment can also be seen as micro foundation for adjustment costs in investment growth rates as suggested by Christiano, Eichenbaum, and Evans (2005).

The main friction in the time-to-build model of Kydland and Prescott (1982) comes from the gestation lag in the law of motion for capital. Since the focus of this paper is investment commitment, my second assumption is a one period lag in the capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t \tag{8}$$

The only free parameter in the commitment friction is w. Because the investment expenditures decrease geometrically over time, the half-life of a project is[9]

$$H = \frac{-\ln(2)}{4\ln(w)} \tag{9}$$

That means, after H years, the firm has incurred half the project's total costs. As a result, w can be calibrated to the data because we can observe the length of projects. In contrast, the free parameter of convex adjustments costs has been subject to much controversy because estimates seem unrealistically large.[10]

3.3 Firm's Optimality Conditions

In this section, I derive the firm's optimality conditions and characterize the firm's value function. The firm's value function is

$$V(K_t, I_{t-1}, Z_t) = \max_{I_t} \{Z_t^{1-\alpha_1} K_t^{\alpha_2} - I_t + \mathbb{E}_t[M_{t+1}V(K_{t+1}, I_t, Z_{t+1})]\} \tag{10}$$

subject to

$$K_{t+1} \geq (1-\delta)K_t + I_t \tag{11}$$

$$I_t \geq wI_{t-1} \tag{12}$$

[9]The project half-life in quarters is defined as time τ when

$$\frac{1-w}{w}(w + w^2 + \dots + w^\tau) = \frac{1}{2}$$

[10]See Gilchrist and Himmelberg (1995) and Erickson and Whited (2000) for recent evidence.

The next three propositions summarize important properties of the firm's value function:

Proposition 1 *There is a unique continuous function $V : \mathcal{K} \times \mathcal{I} \times \mathcal{Z} \to \mathbb{R}$ satisfying (10).*

Proposition 2 *The value function is continuously differentiable in its first and second argument.*

Proposition 3 *For each $Z \in \mathcal{Z}$, $V(\cdot, \cdot, Z) : \mathcal{K} \times \mathcal{I} \to \mathbb{R}$ is strictly increasing (decreasing) in its first (second) argument and strictly concave.*

All proofs are contained in Appendix A. In the following, let V_i denote the derivative of the value function with respect to its i-th element. Propositions 1 and 2 are standard results. Proposition 3 tells us that the value function is decreasing in lagged investment, since higher lagged investment reduces the feasible choice set of the firm. Moreover, the envelope condition with respect to lagged investment is

$$V_2(K_t, I_{t-1}, Z_t) = -w\mu_t$$

where μ_t denotes the multiplier on the commitment constraint (12) and thus measures the shadow costs of commitment. This envelope condition says that the slope of the value function with respect to lagged investment measures the shadow cost of commitment. A binding commitment constraint raises the economic shadow costs μ and causes additional curvature in the value function.

In Appendix A, I show that the firm's optimality conditions are

$$q_t = \mathbb{E}_t[M_{t+1}(Z_{t+1}^{1-\alpha_1}\alpha_2 K_{t+1}^{\alpha_2 - 1} + q_{t+1}(1-\delta))] \tag{13}$$

$$q_t = 1 - \mu_t + w\mathbb{E}_t[M_{t+1}\mu_{t+1}] \tag{14}$$

where q_t denotes the Lagrange multiplier on (11), which is the shadow value of capital and usually termed marginal Q. Equation (13) equates the marginal costs of investment (left side) with the expected marginal benefit of investment (right side). Equation (14) determines marginal Q in terms of the current and expected shadow costs of commitment μ_t and $\mathbb{E}_t[M_{t+1}\mu_{t+1}]$, respectively. In the standard irreversible investment model ($w = 0$), $q_t = 1 - \mu_t$ and marginal Q is falling in the severity of the binding constraint as measured by μ. Commitment gives rise to a second term, $\mathbb{E}_t[M_{t+1}\mu_{t+1}]$, which captures the effect that the commitment constraint is not fixed, but state-dependent. This term is not contained in the irreversible investment model because the lower bound on investment expenditures is constant at zero. It is important to note that this additional term enters positively into the expression for marginal Q and therefore *lowers* it. The reason is that μ measures the economic costs of a binding commitment constraint under the assumption that the constraint is fixed at the current level. Yet the level of the constraint falls at the rate w at which the firm completes initiated projects. This reduction of the commitment level is captured by the term $\mathbb{E}_t[M_{t+1}\mu_{t+1}]$.

Under a linear production function, I can derive an expression for firm value and the stock price in terms of endogenous variables.

Proposition 4 *With a linear production function, $\alpha_2 = 1$, the cum-dividend firm value is given by*

$$V_t = Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1} \tag{15}$$

and ex-dividend firm value, i.e. stock price, is

$$P_t = q_t K_{t+1} - w I_t \mathbb{E}_t[M_{t+1}\mu_{t+1}] \tag{16}$$

Equation (16) of Proposition 4 states that the stock price contains two terms. The first one measures the value of capital and the second one captures the value of committed expenditures. Since $w > 0$ in the commitment model, marginal and average Q deviate from each other, where average Q (or market-to-book ratio) is defined as

$$Q_t = \frac{P_t}{K_{t+1}} = 1 - \mu_t + \frac{K_{t+1} - I_t}{K_{t+1}} w \mathbb{E}_t[M_{t+1}\mu_{t+1}] \tag{17}$$

Equation (17) implies that the market-to-book ratio is less than one when the commitment constraint is binding, $\mu_t > 0$. The last term captures the fact that the commitment constraint is not constant, but state-dependent.

Breaking the equivalence of marginal and average Q has interesting implications for stock return predictability. In the standard irreversible investment model, $w = 0$, the equivalence of marginal and average Q holds. As a result, investment and stock returns are identical state-by-state[11] and given by

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + Q_{t+1}(1-\delta)}{Q_t}$$

[11] The equivalence of investment and stock returns under CRS also holds in the convex adjustment cost model and Liu, Whited, and Zhang (2009) test this implication. Yet it does not hold in the commitment model; see Appendix A for details.

This equation implies that the market-to-book ratio is a sufficient statistic for expected returns since Z_{t+1} follows a random walk. Without an investment friction, the market-to-book ratio is constant and so are expected returns. A time-varying market-to-book ratio causes capital gains in stock returns. These capital gains represent time-variation in the marginal cost of capital, which measure the slope of the firm's value function with respect to current capital, i.e $V_1(K_t, I_{t-1}, Z_t)$.[12]

In the model with commitment, a second source for capital gains in stock returns arises, namely, time-variation in the marginal cost of committed investment expenditures, μ_t. Proposition 4 implies that stock returns are given by

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + q_{t+1}(1-\delta) - i_t w \mu_{t+1}}{Q_t} \tag{18}$$

where $i_t = I_t/K_{t+1}$ is the investment rate. Expected returns are now a function of the marginal cost of capital, i.e., $V_1(K_t, I_{t-1}, Z_t)$, and the marginal cost of commitment, i.e., $V_2(K_t, I_{t-1}, Z_t)$. Since the econometrician does not observe marginal Q, a natural proxy is the market-to-book ratio. Substituting the market-to-book ratio (17) into (18) gives

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + (1-\delta)Q_{t+1} + i_{t+1}(1-\delta)w\mathbb{E}_{t+1}[M_{t+2}\mu_{t+2}] - i_t w \mu_{t+1}}{Q_t} \tag{19}$$

which suggests a relationship between returns and the current book-to-market ratio as well as current and lagged investment rate. The coefficient on the book-to-market ratio measures the effect of the marginal cost of capital on returns, after controlling for the cost of commitment. Controlling for the marginal cost of capital and the current investment rate, the coefficient on the lagged investment rate measures the effect of higher committed expenditures and therefore the marginal cost of investment commitment. In the following, I set $\alpha_1 = \alpha_2$.

4 Household

The representative household maximizes recursive utility over consumption following Epstein and Zin (1989):

$$U_t = \left\{ (1-\beta)C_t^{\rho} + \beta \left(\mathbb{E}_t[U_{t+1}^{1-\gamma}] \right)^{\rho/(1-\gamma)} \right\}^{1/\rho} \tag{20}$$

where C_t denotes consumption, $\beta \in (0,1)$ the rate of time preference, $\rho = 1 - 1/\psi$ and ψ the elasticity of intertemporal substitution (EIS), and γ relative risk aversion. Implicit in the utility function (20) is a CES time aggregator and CES power utility certainty equivalent.

[12] For details, see Equation (25) in Appendix A and note that $q_t = MB_t$.

Epstein-Zin preferences provide a separation of the elasticity of intertemporal substitution and relative risk aversion. These two concepts are inversely related when the agent has power utility. Intuitively, the EIS measures the agent's willingness to postpone consumption over time and this concept is well-defined even under certainty. Relative risk aversion measures the agent's aversion to atemporal risk (across states). Separating these two concepts is crucial for the results of this paper.[13]

The last piece of the model is the household's budget constraint. The household can buy a risky claim on the firm's dividend stream and a risk-free bond such that the return to his portfolio is

$$R_{t+1}^{w} = s_t \frac{P_{t+1} + D_{t+1}}{P_t} + (1 - s_t) R_{t+1}^{f}$$

where R_{t+1}^{w} is the return on wealth, s_t the fraction of wealth invested in the risky asset, P_t the stock price, D_t the dividend payment, and R_t^f the risk-free rate. Then, the budget constraint reads

$$W_{t+1} = (W_t - C_t) R_{t+1}^{w} \tag{21}$$

where W_t denotes wealth.

4.1 Elasticity of Intertemporal Substitution

The magnitude of the EIS plays an important role for the model's implications regarding the dynamics and magnitude of excess returns. Specifically, the commitment constraint binds frequently only when the EIS is large enough. This section therefore contains a discussion on how the EIS affects the consumption policy function.

I show in Appendix B that the consumption-wealth ratio is given by

$$\varphi_t = \frac{A_t}{1 + A_t} \qquad A_t = \left(\hat{\mu}_t^{\rho} \frac{\beta}{1 - \beta} \right)^{1/(\rho - 1)}$$

where $\hat{\mu}_t = \left(\mathbb{E}_t [\phi_{t+1} R_{t+1}^{w}]^{1-\gamma} \right)^{1/(1-\gamma)}$ and ϕ_t is the utility-wealth ratio defined in Equation (20). Because the fraction of wealth which is not consumed has to be invested, this equation also determines the investment policy in real capital.

In Figure 5, I plot the consumption-wealth ratio φ as a function of the EIS for a given value of $\hat{\mu}$. The solid line corresponds to the case when $\hat{\mu} = 0.1$ and the dashed line when

[13] Another important feature of Epstein-Zin preferences is the endogenous preference for early or late resolution of uncertainty.

$\hat{\mu} = 10$. Two effects are notable: First, for a given $\hat{\mu}$, the consumption-wealth ratio is falling in the EIS, implying that a larger fraction of wealth is invested in the risky asset, which here is real capital. More precisely, the consumption-wealth ratio is strictly falling in the EIS if and only if $\ln \hat{\mu} > \ln((1-\beta)/\beta)$. This inequality has an intuitive interpretation. First note that the right hand side of the inequality is approximately the logarithm of the rate of time preference when beta is close to one.[14] Consequently, as long as the log (certainty equivalent) return on wealth is larger than the rate of time preference, an agent who is willing to postpone consumption (high EIS), invests a larger fraction of her wealth in the risky asset.[15]

Second, when the EIS is larger than one, an increase in expected returns, i.e. a jump from the solid to the dashed line, lowers the consumption-wealth ratio because the intertemporal substitution effect dominates the income effect. In a production economy, an increase in expected returns corresponds to a positive technology shock. An agent with an EIS larger than one wants to take advantage of higher productivity and consequently invests a larger fraction of her wealth in capital.

4.2 Household's Equilibrium Conditions

The household's first order condition with respect to s_t gives the standard Lucas Euler equation for stock returns

$$1 = \mathbb{E}_t[M_{t+1} R_{t+1}] \tag{22}$$

where

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \tag{23}$$

denotes the return on equity. Note that in equilibrium the household has to hold all shares, i.e. $s_t = 1$. Further, the risk-free rate asset is in zero net supply and determined by $R^f_{t+1} = 1/\mathbb{E}_t M_{t+1}$.

The pricing kernel based on Epstein-Zin preferences is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho - 1} \left(\frac{U_{t+1}}{\left(\mathbb{E}_t [U_{t+1}^{1-\gamma}] \right)^{1/(1-\gamma)}} \right)^{1-\gamma-\rho} \tag{24}$$

An important property of this pricing kernel is its dependence on the agent's value function. Intuitively, since the value function is the present value of future utility from consumption,

[14]The rate of time preference is usually set around 0.99.

[15]See Bhamra and Uppal (2006) for more details on Epstein-Zin preferences.

an agent with Epstein-Zin preferences cares about her future consumption path and the value function measures the agent's satisfaction thereof. Because the agent cares about future consumption, shocks to future consumption growth are priced. In contrast, when the agent has power utility, the pricing kernel depends only on marginal utility. Accordingly, the agent evaluates risks at different dates in isolation and shocks to the consumption path are not priced.

5 Model Results

5.1 Calibration

In this section, I explain the calibration of the model. The model is solved numerically as described in Appendix C. Table 2 summarizes the parameter choices. These values are similar to Cooley and Prescott (1995) and Boldrin, Christiano, and Fisher (2001) and correspond to a quarterly frequency. The growth rate of the economy g is set to 0.4%. The capital share of production is $\alpha = 0.4$. The quarterly depreciation rate of capital δ is set to 2% implying 10% annual depreciation. The innovations to the technology shock, ε_t, are mean zero with standard deviation σ of 0.03 percent.

The last parameter of the calibration is the commitment parameter w. As shown with Equation (9), w determines the half-life of a project. The empirical investment literature makes it possible to calibrate w because it contains results regarding the duration of investment projects. Several papers have estimated the length of investment projects; however, estimates vary strongly across industries. For instance, Mayer (1960) conducts a survey of 110 companies and finds that the average length of the time between the decision to build a plant and its completion is 21 months. MacRae (1989) notes that investments in a power generating plant typically takes 6 to 10 years to complete; Pindyck (1991) observes that investment lags in the aerospace and pharmaceutical industries are comparable. In a more recent study, Koeva (2000) reports lead times between 13 months for the rubber industry and 86 months for utilities. Based on the empirical evidence, I choose a project half-life of $H = 3.5$ years, implying $w = 0.95$.

5.2 Stock Market Moments

Figure 6 shows the consumption policy function for the base case parameterization when the shock $Z_t = 1$. The commitment constraint is clearly visible as a kink in the consumption policy function since a binding commitment constraint imposes an upper bound on consumption. Specifically, when the economy is hit by an adverse shock, the agent would like to lower investment, but a binding commitment constraint prevents the firm from doing so. Consequently, consumption falls by more than is optimal.

In Table 3, I present numerical results for the stock market. I simulate the economy for 5,000 periods and report unconditional moments. Instead of presenting results only for the full model, I consider 4 cases: low and high EIS, as well as commitment and no commitment. I do not report results for the power utility case because the mean and the volatility of the risk-free rate are unrealistically large. This comparison facilitates a better understanding of each model component. In Model A and B, the agent has low and high EIS, respectively, but there is no investment commitment. Model C and D contain commitment and the agent has low and high EIS, respectively. Following Bansal and Yaron (2004), I choose risk aversion of 10 and EIS of 0.5 and 1.5.

Model A and B do not generate realistic first and second moments of stock returns, which is the equity premium puzzle. Combining the commitment friction with a low EIS agent (Model C) still does not generate a large risk premium because the commitment constraint is seldom binding. The reason is that after a positive shock an agent with a low EIS does not invest much because of the strong desire to smooth consumption over time. With low investment in good times, the commitment constraint does not increase much and therefore binds occasionally in recessions. In contrast, an agent with a high EIS wants to take advantage of higher productivity and is willing to accept more variation of consumption growth over time as explained in Section 4.4.1. As a result, the commitment constraint rises dramatically following good productivity, leading to a commitment constraint that binds more frequently when the economy switches into a recession. As a result, the full model (Model D) generates a risk premium of 3.5%—a marked increase over the model without commitment (Model B) and low EIS (Model C).

The equity premium generated by the full model (D) is lower than common estimates of around 6%. What explains this discrepancy? First, in the real world, equity is a leveraged claim on the firm's profit and this model does not contain financial leverage. Following Gomes, Kogan, and Yogo (2009), the data implies a leverage-free risk premium of only 3.5% which is arguably a better benchmark for my model.[16] Accordingly, Bhamra, Kuehn, and Strebulaev (2010) show that a significant portion of the equity premium can be explained with financial leverage. Second, using the Gordon growth model, Fama and French (2002) find that the average expected equity premium for the period 1872-2000 is 3.5%. By the law of large numbers, the average expected and average realized excess return should converge in the long run. However in finite samples, this does not have to hold. More importantly, Fama and French (2002) argue that the equity premium estimate based on the Gordon growth model is closer to the true expected value than the estimate from average returns because of smaller standard errors of the former.

The finding that the EIS has a strong impact on the equity premium contrasts with Tallarini (2000) who finds that only risk aversion, but not the EIS, affects the risk premium. I reach the opposite conclusion because his model does not contain any investment friction.

A high EIS is also necessary to achieve a realistic risk-free rate volatility. In Models A and C, the volatility of the risk-free rate is at least twice as high as in the data. A high risk-free rate volatility is also the major shortcoming of internal habit based explanations of the equity premium puzzle such as Jermann (1998) and Boldrin, Christiano, and Fisher (2001). Internal habit preferences increase the curvature of the utility function, thereby raising risk aversion and lowering the EIS. A low EIS means that the household is very eager to smooth consumption over time. To achieve this goal, the household's demand for the risk-free bond is high, especially in recessions. But the supply is perfectly inelastic, since the bond is in zero net supply. To accommodate these demand swings, the risk-free rate has to adjust accordingly, resulting in a large risk-free rate volatility.

Even though Campanale, Castro, and Clementi (2010) rely on recursive preferences, their choice of a low EIS has the same counterfactual implication regarding the risk-free rate volatility. In the commitment model (D) with a high EIS, however, the volatility of the risk-free

[16] The expected return of a leveraged portfolio, which is long V dollars in the market portfolio and short bV dollars in the risk-free asset, is given by

$$\mathbb{E}[R^L] = \frac{1}{1-b}\mathbb{E}[R] - \frac{b}{1-b}R^f$$

where R^L is the levered market return. Using US data as reported in Table 3 and a leverage ratio of $b = 0.52$ for the post-war sample (Gomes, Kogan, and Yogo (2009)) implies that the leverage-free risk premium $\mathbb{E}[R] = 3.5\%$.

rate is 0.89% compared to 0.97% in the data. Thus, the magnitude of the equity premium is not achieved at the cost of a high risk-free rate volatility.

The Sharpe ratio, $\mathbb{E}[R-R^f]/\sigma(R-R^f)$, is lowest in the low EIS model without commitment (A) and highest in the full model (D). In high EIS commitment model, it is 0.37, compared to 0.33 in the data. In the last row of Table 3, I report the volatility of time aggregated annual consumption growth, $\sigma(\Delta \ln C^a)$. It varies between 2.1% and 2.9%, compared to 2.9% in the data. The reason for the equity premium puzzle is the low volatility of consumption growth. It is therefore important that models, which try to explain the equity premium, do not exceed empirical estimates of the consumption volatility.

As a robustness check, I also compute the equity premium for other project half-lifes. The equity premium increases in the half-life of projects because the frequency of a binding commitment constraint increases, too. When the project half-life is only 2 years, the equity premium falls to 2.8%. When the project half-life increase to 5 years, the equity premium reaches 4%.

5.3 Consumption Volatility

A negative technology shock causes a negative shock to current consumption growth and leads to an increase in the volatility of consumption growth when the commitment constraint binds. But more importantly, the effect of investment commitment on consumption is not restricted to the instantaneous impact but is long-lasting. In particular, I will show that the conditional consumption volatility is high for several periods after the adverse shock. This evidence illustrates that the commitment friction provides a strong internal propagation mechanism, in particular, since the model is driven by a (geometric) random walk.

To gauge the strong internal propagation mechanism, I first estimate a GARCH(1,1) process for consumption data to have a measure of the conditional consumption volatility. In simulated data, a binding commitment constraint can be identified by a high book-to-market ratio as shown with Equation (17).

In Panel A of Table 6, I report estimates of a GARCH(1,1) process for consumption growth based on real data and simulated data using quasi maximum likelihood. A GARCH(1,1) process postulates the following process for the conditional variance of log consumption growth

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2$$

where ε_t is the demeaned consumption growth rate. Consumption is real non-durable plus service expenditures at quarterly frequency for the period 1947-2006. In the data, both the ARCH component w_1 and the GARCH component w_2 are significant at the 1% level. The model generates a similar result. The volatility of consumption growth is explained well by a GARCH(1,1) process, but with a higher coefficient on the ARCH component w_1 and lower one on the GARCH component w_2. This means that the instantaneous effect is stronger in the model than in the data and the consumption volatility is more persistent in the data than in the model.

Panel B reports forecasting regressions of future consumption volatilities, σ_{t+s}, on the current log book-to-market ratio BM_t using the estimated time-series from the GARCH(1,1). As a proxy for the aggregate book-to-market ratio I use the average value-weighted book-to-market ratio taken from Kenneth French's website.[17] I use the quarterly return of the value-weighted CRSP index to compute quarterly book-to-market ratios. In the data, the current book-to-market ratio is positively related with current consumption volatility, implying that consumption volatility is counter-cyclical. Further, a high book-to-market ratio forecasts a high consumption volatility over the next several quarters.[18] The model generates similar results. A binding commitment constraint causes an increase in the book-to-market ratio and an increase in consumption volatility. Similar to the data, a high book-to-market ratio also causes high consumption volatility for the next quarters. This implies that the effect of a binding commitment constraint is not momentary, but instead causes a *persistent* shock to the volatility of the equilibrium consumption process. This mechanism is exactly the second building block of the long-run risk of Bansal and Yaron (2004) and the reason why the commitment model outperforms the standard production economy without commitment. In the model, the effect is again less persistent than in the data.

Panel C presents forecasting regressions of the future log book-to-market ratio BM_{t+s} on the current consumption volatility σ_t. The data shows that a rise in consumption volatility predicts a fall in asset prices. Bansal and Yaron (2004) show analytically that an EIS greater than one is necessary for this relation to hold—a condition satisfied in the model. In line with

[17] Specifically, I use the data of the 3-portfolio sort and weight the average value-weighted book-to-market ratio of each tercile with their respective weights of 30%, 40% and 30% to get a proxy for the aggregate book-to-market ratio.

[18] This evidence is consistent with Kandel and Stambaugh (1990), Bansal, Khatchatrian, and Yaron (2005) and Bansal and Yaron (2004) who use the price-dividend and price-earnings ratio.

the data, high consumption volatility forecasts a high book-to-market ratio.

5.4 Cyclicality and Persistence

This section examines the cyclicality and persistence of moments of returns and consumption growth. To gain a a better understanding of the cyclicality of stock returns, I rewrite the Euler equation (22) in terms of the risk-free rate, R^f_{t+1}, the conditional correlation between the pricing kernel and future stock returns, $\text{Corr}_t(M_{t+1}, R_{t+1})$, the conditional volatility of the pricing kernel, $\sigma_t(M_{t+1})$ and the conditional volatility of stock returns, $\sigma_t(R_{t+1})$, i.e.

$$\mathbb{E}_t[R^e_{t+1}] \quad = \quad -R^f_{t+1}\text{Corr}_t(M_{t+1}, R_{t+1})\sigma_t(M_{t+1})\sigma_t(R_{t+1})$$

The main determinants of the cyclicality of stock returns are the conditional volatility of the pricing kernel and the conditional volatility of stock returns. Less important is the effect of the risk-free rate; it tends to be procyclical because it is mainly governed by the expected growth rate of consumption. The conditional correlation between the pricing kernel and future stock returns is irrelevant because the model is driven by a single shock and thus these two variables are conditionally perfectly (negatively) correlated.

The standard real capital friction is convex adjustment costs, employed, for instance, by Jermann (1998), Kaltenbrunner and Lochstoer (2010), and Campanale, Castro, and Clementi (2010). A major drawback of this friction is that excess returns tend to be *procyclical*. The problem is that convex adjustment costs have opposing effects on the conditional volatility of the pricing kernel and stock returns.

On the one hand, convex adjustment costs can potentially give rise to countercyclical consumption growth volatility.[19] As productivity falls, investment must be reduced by increasing rates if consumption were to fall at a constant rate. However, convex adjustment costs prevent this from happening, resulting in countercyclical consumption growth volatility. Accordingly, the volatility of the pricing kernel is countercyclical, leading to countercyclical excess returns.

On the other hand, convex adjustment costs also result in procyclical stock return volatility. The reason is the following: Under convex costs, firms incur adjustment costs when they disinvest as well as invest. At the aggregate and sectoral level, investment is always positive. Hence, convex costs impact quantities and prices in booms, leading to procyclical stock return

[19] A point made by Gomes, Kogan, and Zhang (2003) and Cooper and Priestley (2005).

volatility.[20] The second channel usually dominates the first one because the volatility of stock returns is an order of magnitude larger than the volatility of the pricing kernel. Hence, excess returns tend to be procyclical. The investment commitment friction overcomes this drawback because it impacts quantities and prices only in recessions. Consequently, the volatility of the pricing kernel and stock returns are both countercyclical and therefore excess returns are as well.

To assess the cyclicality of the model, Table 4 reports the (unconditional) correlation of realized log consumption growth, $\Delta \ln C_t$, with the conditional volatility of consumption growth, $\sigma_t(\Delta \ln C_{t+1})$, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return variance, $\mathrm{Var}_t(R_{t+1})$, conditional consumption beta, β_t, and market price of risk, λ_t. This table shows the cyclicality for all four model versions as in Table 3. Models A and B have no commitment whereas Models C and D have commitment; Models A and C have a low EIS whereas Models B and D have a high EIS.

In the models without commitment (Model A and B) the conditional volatility of consumption growth, conditional excess returns and the conditional volatility of stock returns are procyclical—contradicting empirical facts. Conditional consumption betas and the market price of risk are procyclical as well.

In the models with commitment (Model C and D) the conditional volatility of consumption growth, conditional excess returns and the conditional volatility of stock returns are countercyclical, consistent with the empirical evidence in Kandel and Stambaugh (1990). Conditional consumption betas are also countercyclical. The market price of risk is only slightly countercyclical. These countercyclicalities are caused by the asymmetric nature of the commitment friction.

Table 5 reports the (unconditional) autocorrelation of the price-earnings ratio, P_t/E_t, the book-to-market ratio, K_{t+1}/P_t, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return volatility, $\sigma_t(R_{t+1})$, conditional consumption beta, β_t, expected consumption growth, $E_t[\Delta \ln C_{t+1}]$, and conditional variance of consumption growth, $\mathrm{Var}_t(\Delta \ln C_{t+1})$ of the commitment model.

Two interesting results emerge: First, in the model, price-scaled ratios, such as the price-dividend and book-to-market ratio, are highly persistent. The same finding applies to the

[20] In a convex adjustment costs model with constant returns to scale, marginal Q equals average Q, i.e. $P_t = q_t K_{t+1}$.

conditional first and second moments of stock returns. Second, conditional first and second moments of consumption growth are also highly persistent, even though the economy is driven by a random walk shock. In their long-run model, Bansal and Yaron (2004) assume that the first-order autocorrelation of expected consumption growth and variance of consumption growth is 0.94 and 0.96, respectively. The commitment model generates an autocorrelation of 0.51 and 0.49, respectively. It therefore provides partial justification of their exogenously assumed consumption process.

5.5 Predictability

The predictability of stock returns is another important feature of the data. As shown in Section 3, in the standard irreversible investment model the book-to-market ratio is a sufficient statistic for expected returns. In the investment commitment model, the book-to-market is not a sufficient statistic for returns, thus giving rise to additional predictor variables. Specifically, since investment is not completed instantaneously, lagged investment arises as an additional state variable in the model. Accordingly, the lagged investment rate helps to forecast returns. The model predicts that, controlling for the current book-to-market ratio and investment rate, there is *positive* relation between the lagged investment rate and the first and second moments of future returns. Higher lagged investment means that the firm has initiated large investment projects in the past which it is committed to complete in the future.

Table 8 reports time-series regressions of the conditional excess return, $\mathbb{E}_t[R^e_{t+1}]$ (Panel A), conditional volatility of stock returns, $\mathrm{Var}_t(R_{t+1})^{1/2}$ (Panel B), and future realized excess returns, R^e_{t+1} (Panel C) on the book-to-market ratio, and current and lagged investment rate. I simulate 50 years of data 100 times and report cross-simulation averages. On simulated data (Regression 1), the book-to-market ratio is positively related with expected returns (Panel A) and realized excess returns (Panel C). Further, the current investment rate (Regression 2) forecasts lower expected and realized excess returns. Panel B shows that future stock return volatility is positively related to the book-to-market ratio (Regression 1) and negatively to the current investment rate (Regression 2).

In univariate Regression 3 of Panel A, B and C, the lagged investment rate is not significant at the 5% level. But Regressions 4 and 6 in Panel A, B, and C confirm the intuition that lagged investment is positively related with future returns and return volatility after controlling for the current book-to-market ratio and/or current investment rate.

To test these predictions, I use the following quarterly data: Excess returns are the difference between the return on the value-weighted CRSP index and the 90-day treasury bill; the investment rate is constructed using (8) based on real nonresidential investment data coming from the BEA-NIPA. Table 9 reports the results. In Panel A, I regress future realized excess returns on the current and lagged investment rate. As a proxy for the conditional return volatility, I use the absolute value of excess returns which I regress on the same predictor variables in Panel B. In both Panels, the lagged investment rate is positively related with future returns and return volatility; yet it is only significant at the 5% level for return volatility.

Another aspect is return predictability at long horizons. Challenging the view that stock returns follow a random walk, Campbell and Shiller (1988) and Fama and French (1988) show that the dividend yield forecasts stock returns and the explanatory power increases with the horizon. I replicate their finding within the model and Table 7 reports the results. Consistent with empirical facts, a high price-earnings ratio predicts lower future returns and the R^2 of the regressions is increasing with the horizon. Previous production economy models, such as Jermann (1998) and Boldrin, Christiano, and Fisher (2001), have not been able to replicate this empirical fact.

6 Conclusion

In this paper, I explore the asset pricing implications of investment commitment in a general equilibrium economy, where the household has Epstein-Zin preferences. A common assumption in literature is that investment occurs instantaneously. A more realistic assumption is that investment projects last for many periods and require current expenditures as well as commitment to future expenditures. The contribution of this paper is to provide a tractable specification of investment commitment and gauge its general equilibrium effects on asset prices.

In equilibrium, consumption and investment are determined jointly and, as a result, the investment commitment friction impacts the equilibrium consumption process. With standard convex or non-convex investment frictions, investment occurs instantaneously and the impact of the friction is momentary. In contrast, investment commitment also impacts the *distribution* of *future* consumption growth rates, because commitments in long-term projects are not satisfied immediately. As a consequence, investment commitment generates time-varying first and second moments of consumption growth. Since the household has Epstein-Zin preferences, this effect gets priced, leading to a significant larger equity premium and return volatility. The same mechanism underlies the long-run risk model of Bansal and Yaron (2004). Whereas they assume an exogenous consumption process with time-varying first and second moments, these dynamics arise endogenously in a production economy with investment commitment.

Investment commitment is also an asymmetric friction. It affects consumption mainly after adverse shocks, when firms would like to reduce investment. Consequently, first and second moments of expected excess returns are endogenously counter-cyclical relative to consumption growth. Furthermore, the model generates novel empirical implications regarding the predictability of stock returns which find support in the data. Lagged investment arises as state variable in the model, capturing the amount of committed expenditures. The model predicts that, ceteris paribus, times with a higher lagged investment rate are riskier because the consequences of adverse shocks are more severe. Using aggregate data, I find that there is a positive relation between the lagged investment rate and future returns and return volatility which is significant for the latter.

Appendix

A Firm

Optimality conditions: Optimal firm behavior can be characterized by studying the firm's first-order conditions. The first-order conditions with respect to K_{t+1} and I_t and the Kuhn-Tucker condition are

$$\begin{aligned} -q_t + \mathbb{E}_t M_{t+1} V_1(K_{t+1}, I_t, Z_{t+1}) &= 0 \quad (25) \\ -1 + q_t + \mu_t + \mathbb{E}_t M_{t+1} V_2(K_{t+1}, I_t, Z_{t+1}) &= 0 \quad (26) \\ \mu_t (I_t - w I_{t-1}) &= 0 \end{aligned}$$

where q_t is the multiplier on (11) and thus the shadow value of capital usually termed marginal Q. μ_t is the multiplier on the commitment constraint (12) and thus the shadow costs of commitment. The Kuhn-Tucker condition guarantees that either $\mu_t \geq 0$ or $(I_t - wI_{t-1}) \geq 0$ holds.

The envelope conditions with respect to the two endogenous state-variables, current capital and lagged investment expenditures, are

$$\begin{aligned} V_1(K_t, I_{t-1}, Z_t) &= Z_t^{1-\alpha_1} \alpha_2 K_t^{\alpha_2 - 1} + q_t(1-\delta) \quad (27) \\ V_2(K_t, I_{t-1}, Z_t) &= -w\mu_t \quad (28) \end{aligned}$$

Combining the first-order conditions (25) and (26) with the envelope conditions (27) and (28) gives the Equations (13) and (14) in the main text.

Proofs: In the proofs, I drop the time index t and denote next period's variables with a $'$ and last period's variable with a $^-$. For the proofs of Proposition 1-2, I need the following assumptions: Let the shock $Z \in \mathcal{Z} = [\underline{Z}, \bar{Z}]$ and the transition function Q satisfies the Feller property. Let the capital stock $K \in \mathcal{K} = [0, \bar{K}]$ where $\bar{K}$ solves $\bar{Z}\,\bar{K}^{\alpha} - \delta = 0$. Let investment $I \in \mathcal{I} = [0, \bar{K}]$.

The feasible policy correspondence is

$$\Gamma(K, I^-) = \{(K', I) : K' \in \mathcal{K} \text{ and } I \in [wI^-, \bar{K}]\}$$

Proof of Proposition 1: The set $\mathcal{K} \times \mathcal{I}$ is non-empty, compact and convex. Because F is continuous with compact domain, it is bounded. Note that $\Gamma(K, I^-)$ is nonempty, compactvalued, and continuous. Thus the Assumptions 9.4-9.7 in Stokey and Lucas (1989) (hereafter SL) are satisfied. The proposition follows from Theorem 9.6 in SL.

Proof of Proposition 2: From Theorem 9.10 in SL follows that for each (K', I) in the interior of $\mathcal{K} \times \mathcal{I}$, V is continuously differentiable. Sargent (1980) extends this result to corner solutions.

Proof of Proposition 3: To prove monotonicity of the value function, simply redefine the problem in terms of $\bar{I} = -I$. The redefined value function satisfies $v(K, \bar{I}^-, Z) = V(K, I^-, Z)$. The new correspondence $\Gamma(K, \bar{I}^-) = \{(K', \bar{I}) : K' \in \mathcal{K} \text{ and } \bar{I} \in [-\bar{K}, -w\bar{I}^-]\}$ is increasing in $(K', \bar{I})$. Thus the Assumptions 9.4-9.9 in SL are satisfied. The monotonicity of v follows from Theorem 9.7 in SL which implies that V is strictly increasing (decreasing) in its first (second) argument.

Because $\alpha \in (0,1)$, F is strictly concave. Because $\mathcal{K} \times \mathcal{I}$ is convex, so is Γ. Thus, Assumption 9.10-9.11 in SL are satisfied. The concavity of V follows from Theorem 9.8 in SL.

Proof of Proposition 4: Firm value can also be stated in the Lagrange form

$$\begin{aligned} V_t &= Z_t^{1-\alpha_1} K_t - I_t + q_t((1-\delta)K_t + I_t - K_{t+1}) + \mu_t(I_t - wI_{t-1}) \\ &+ \mathbb{E}_t M_{t+1} \left\{ Z_{t+1}^{1-\alpha_1} K_{t+1} - I_{t+1} + q_{t+1}((1-\delta)K_{t+1} + I_{t+1} - K_{t+2}) + \mu_{t+1}(I_{t+1} - wI_t) \right\} + ... \end{aligned}$$

To simplify the Lagrange equation, multiply (13) with K_{t+1} and (14) with I_t

$$\begin{aligned} q_t K_{t+1} &= \mathbb{E}_t M_{t+1}(Z_{t+1}^{1-\alpha_1} + q_{t+1}(1-\delta))K_{t+1} \\ q_t I_t &= I_t - \mu_t I_t + w\mathbb{E}_t M_{t+1}\mu_{t+1} I_t \end{aligned}$$

Next plug the modified first-order conditions into the Lagrange function. Firm value then simplifies to

$$V_t = Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1}$$

The firm's stock price is

$$\begin{aligned} P_t &= V_t - D_t \\ &= Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1} - (Z_t^{1-\alpha_1} K_t + (1-\delta)K_t - K_{t+1}) \\ &= q_t K_{t+1} - wI_t \mathbb{E}_t M_{t+1}\mu_{t+1} \end{aligned}$$

which proves the claim.

Investment vs. Stock Returns: Investment commitment also causes a wedge between investment and stock returns. To see that the equivalence here does not hold, note that equation (13) defines the investment return. With a linear production function, the investment return simplifies to

$$R^I_{t+1} = \frac{Z^{1-\alpha_1}_{t+1} + q_{t+1}(1-\delta)}{q_t} \tag{29}$$

The investment return R^I_{t+1} defines the firm's intertemporal tradeoff of investing one more unit of capital. It is the ratio of tomorrow's marginal benefits of investing one additional unit of capital divided by today's marginal costs.

By contrast, stock returns are

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{Z^{1-\alpha_1}_{t+1} K_{t+1} + q_{t+1}(1-\delta)K_{t+1} - \mu_{t+1} I_{t+1}}{q_t K_{t+1} - w I_t \mathbb{E}_t M_{t+1} \mu_{t+1}}$$

The difference between investment and stock returns arises because the former reflects only the tradeoff between marginal costs and benefits of *new* investment projects. The latter, however, is the return to the entire firm value which also includes the value of committed expenditures. Breaking the equivalence between investment and stock returns is important for explaining stock returns since investment returns are tied to the intertemporal tradeoff of real capital which does not fluctuate much.

B Household

The homogeneity of the utility function (20) and the linearity of the budget constraint (21) imply that U is also homogeneous (scale invariant) in wealth, i.e.

$$U(W_t, \xi_t) = \phi(\xi_t) W_t = \phi_t W_t \tag{30}$$

where ξ_t is a vector of state variables coming from the production side of the economy. For the same reason, the consumption policy function is proportional in wealth, i.e. $C_t = \varphi(\xi_t) W_t = \varphi_t W_t$, where φ_t is the consumption-wealth ratio.

Substituting the guess for U (30) and the budget constraint (21) into the utility function (20) yields

$$U(W_t, \xi_t) = \max_{C_t} \left\{ (1-\beta) C_t^\rho + \beta \hat{\mu}_t^\rho (W_t - C_t)^\rho \right\}^{1/\rho}$$

where $\hat{\mu}_t = \mathbb{E}_t[(\phi_{t+1} R^w_{t+1})^\alpha]^{1/\alpha}$.

The first-order condition with respect to consumption C_t is

$$(1-\beta)C_t^{\rho-1} = \beta\hat{\mu}_t^{\rho}(W_t - C_t)^{\rho-1}$$

Combing the first-order condition with the guess for the consumption policy function gives the solution for the consumption-wealth ratio

$$\varphi_t = \frac{A_t}{1+A_t} \qquad A_t = \left(\hat{\mu}_t^{\rho}\frac{\beta}{1-\beta}\right)^{1/(\rho-1)}$$

The derivative of the consumption-wealth ratio with respect to the EIS is

$$\frac{\partial\varphi}{\partial\psi} = -\frac{((1-\beta)\beta)^{\psi}\mu^{1+\psi}(\ln\mu - \ln(1-\beta) + \ln\beta)}{((1-\beta)^{\psi}\mu + \beta^{\psi}\mu^{\psi})^2}$$

which is positive if and only if $\ln\mu - \ln(1-\beta) + \ln\beta > 0$.

C Numerical Solution Method

The model is solved by imposing a competitive equilibrium and market clearing. The following definition states this precisely.

Definition 1 *A competitive rational expectations equilibrium is a sequence of allocations* $\{C_t, K_t\}_{t=0}^{\infty}$ *and a price system* $\{\Lambda_t, P_t\}_{t=0}^{\infty}$ *such that: (i) given the price system, the representative household maximizes (20) s.t. (21); (ii) given the price system, the representative firm maximizes (10) s.t. (11) and (12); (iii) the good market clears* $Y_t = C_t + I_t$*; (iv) the stock market clears* $s_t = 1$*.*

Because of the welfare theorems, the decentralized economy and the social planner give the same optimal allocations. Since solving the latter is computationally easier, I first solve the planner's problem for optimal quantities. Second, given quantities I use the Euler equation to solve for prices.

The planner's problem is

$$J(K_t, I_{t-1}, Z_t) = \max_{C_t}\left\{(1-\beta)C_t^{\rho} + \beta\mathbb{E}_t[J(K_{t+1}, I_t, Z_{t+1})^{\alpha}]^{\rho/\alpha}\right\}^{1/\rho}$$

s.t.

$$
\begin{aligned}
Z_t^{1-\alpha} K_t^\alpha &= C_t + I_t \\
K_{t+1} &= (1-\delta) K_t + I_t \\
I_t &= (1-w) X_t + w I_{t-1} \\
X_t &\geq 0 \\
\frac{Z_{t+1}}{Z_t} &= e^{\Delta z_{t+1}} = e^{(1-\varrho)g + \varrho \Delta z_t + \sigma \varepsilon_{t+1}}
\end{aligned}
$$

Because the technology follows a geometric random walk, the problem has to be reformulated so that it is difference stationary. Stationary variables are denoted with a hat, i.e., $\hat{C}_t = C_t/Z_t$, $\hat{K}_t = K_t/Z_t$, $\hat{I}_t = I_t/Z_t$, $\hat{X}_t = X_t/Z_t$, $\hat{J}_t = J_t/Z_t$.

Define committed investment in current productivity units by

$$
\hat{V}_t = e^{-\Delta z_t} \hat{I}_{t-1} \qquad \hat{V}_{t+1} = e^{-\Delta z_{t+1}} \hat{I}_t
$$

The stationary planner's problem is

$$
\hat{J}\left(\hat{K}_t, \hat{V}_t, \Delta z_t\right) = \max_{\hat{C}_t} \left\{ (1-\beta)\hat{C}_t^\rho + \beta \mathbb{E}_t \left[e^{\alpha \Delta z_{t+1}} \hat{J}\left(\hat{K}_{t+1}, \hat{V}_{t+1}, \Delta z_{t+1}\right)^\alpha \right]^{\rho/\alpha} \right\}^{1/\rho}
$$

s.t.

$$
\begin{aligned}
\hat{K}_t^\alpha &= \hat{C}_t + \hat{I}_t \\
\hat{K}_{t+1} &= \big((1-\delta)\hat{K}_t + \hat{I}_t\big) e^{-\Delta z_{t+1}} \\
\hat{I}_t &= (1-w)\hat{X}_t + w \hat{V}_t \\
\hat{X}_t &\geq 0 \\
\Delta z_{t+1} &= (1-\varrho) g + \varrho \Delta z_t + \sigma \varepsilon_{t+1}
\end{aligned}
$$

After having solved for quantities, I use the Euler equation to solve for the equilibrium price function $P_t = P(K_t, I_{t-1}, Z_t)$

$$
P_t = \mathbb{E}_t M_{t+1} (D_{t+1} + P_{t+1})
$$

As before, define the stationary variable $\hat{D}_t = D_t/Z_t$ and $\hat{P}_t = P_t/Z_t$. The Euler equation in terms of stationary variables is

$$
\hat{P}_t = \mathbb{E}_t M_{t+1} e^{g+\varepsilon_{t+1}} (\hat{D}_{t+1} + \hat{P}_{t+1})
$$

The model is solved on a 3-dimensional discrete grid. The aggregate shock ε_t is modeled as a Markov chain with 5 states. The grid for capital and lagged investment has 80 and 40 elements, respectively. The planner's value function is solved with modified policy iteration and bicubic interpolation for a choice vector with 5,000 elements. Given the value function and optimal allocations, the stationary price functional is solved numerically as a fixed point problem on the same grid as the value function with bicubic interpolation.

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Figure 1: Annualized Consumption Growth Volatility

This figure plots the EGARCH volatility estimates for real non-durable plus service consumption expenditure growth. The data is quarterly and covers the period form 1952 to 2011. Grey bars represent NBER recessions.

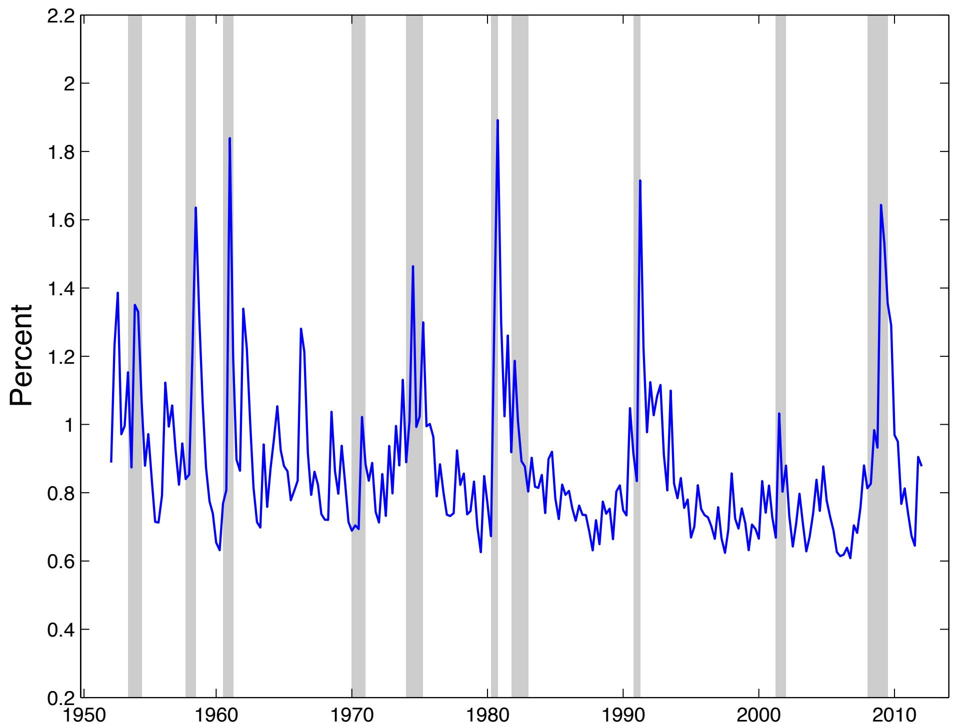

Figure 2: Annualized Consumption Growth Volatility

This figure plots the EGARCH volatility estimates for real personal consumption expenditure growth. The data is quarterly and covers the period form 1952 to 2011. Grey bars represent NBER recessions.

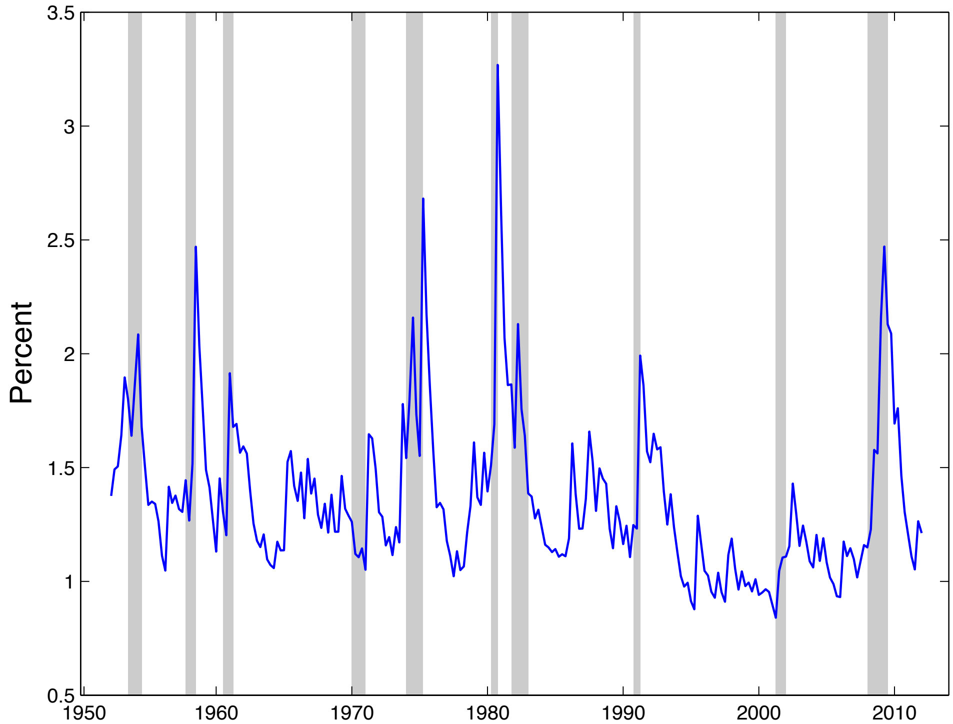

Figure 3: Annualized Output Growth Volatility

This figure plots the EGARCH volatility estimates for output growth. he data is quarterly and covers the period form 1952 to 2011. Grey bars represent NBER recessions.

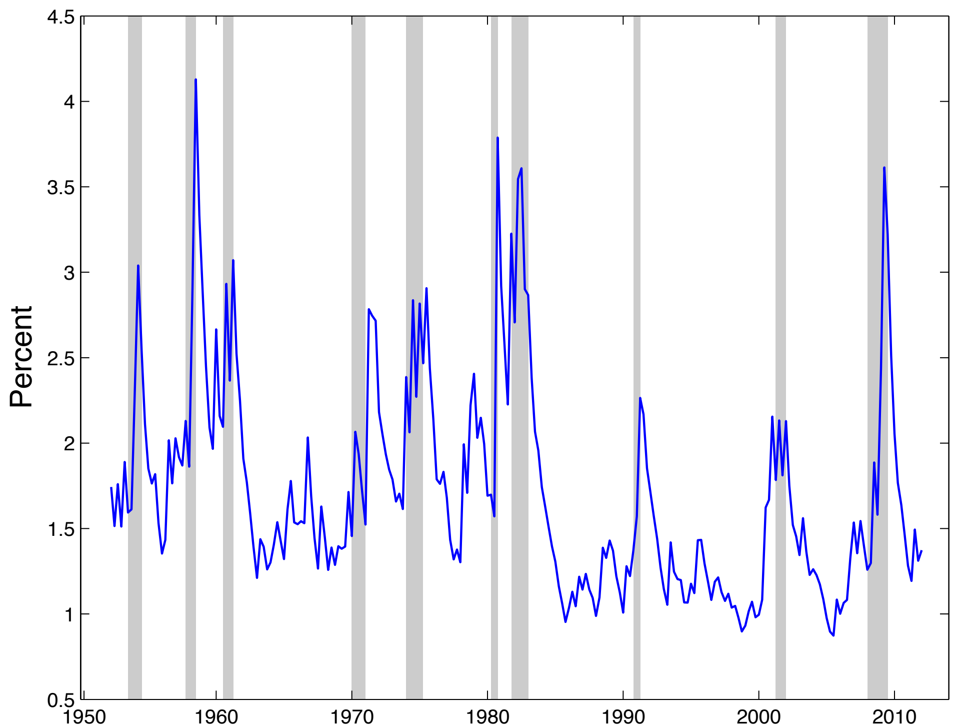

Figure 4: Annualized Stock Return Volatility

This figure plots the EGARCH volatility estimates for aggregate stock returns as measured by the value-weighted CRSP index. The data is quarterly and covers the period form 1952 to 2011. Grey bars represent NBER recessions.

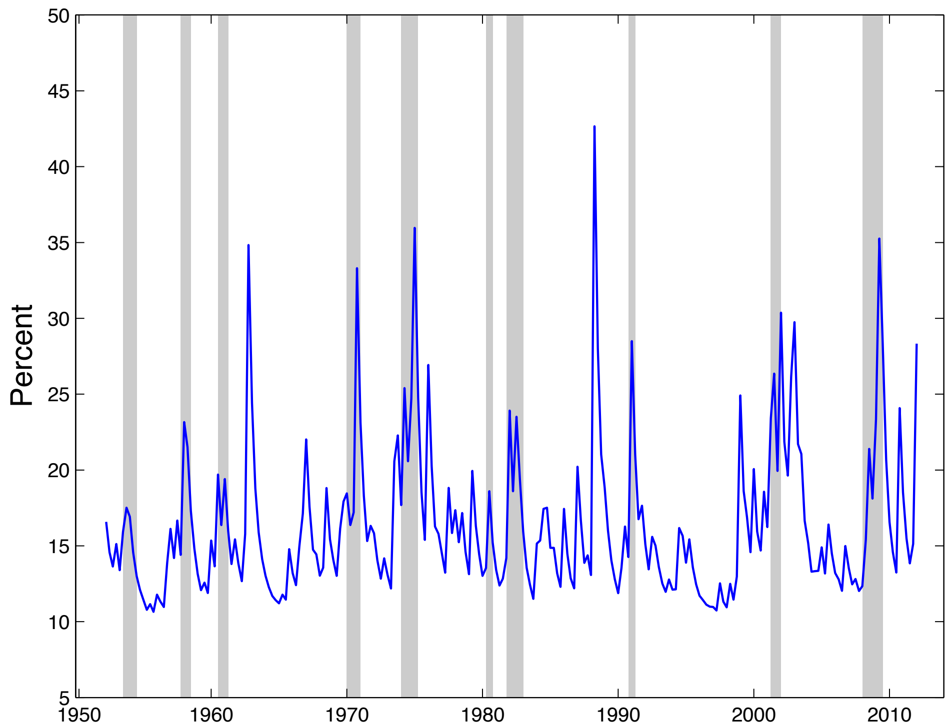

Figure 5: Consumption-Wealth Ratio

This figure depicts the consumption-wealth ratio φ as a function of EIS for a constant $\hat{\mu}$. The solid line represents $\hat{\mu} = 0.1$ and the dashed one $\hat{\mu} = 10$.

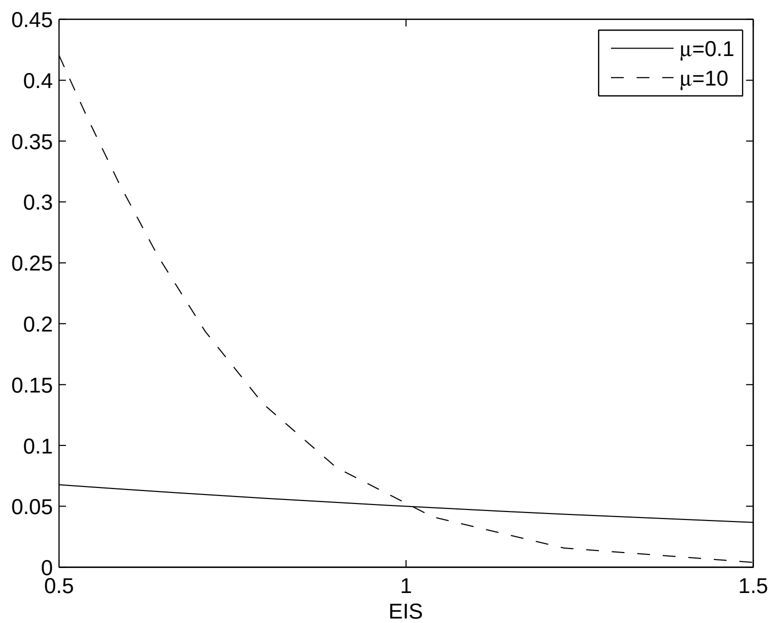

Figure 6: Consumption policy function

This figure shows the consumption policy function, $C_t = C(K_t, I_{t-1}, Z_t)$, as a function of capital, K_t, and lagged investment, I_{t-1}, at $Z_t = 1$.

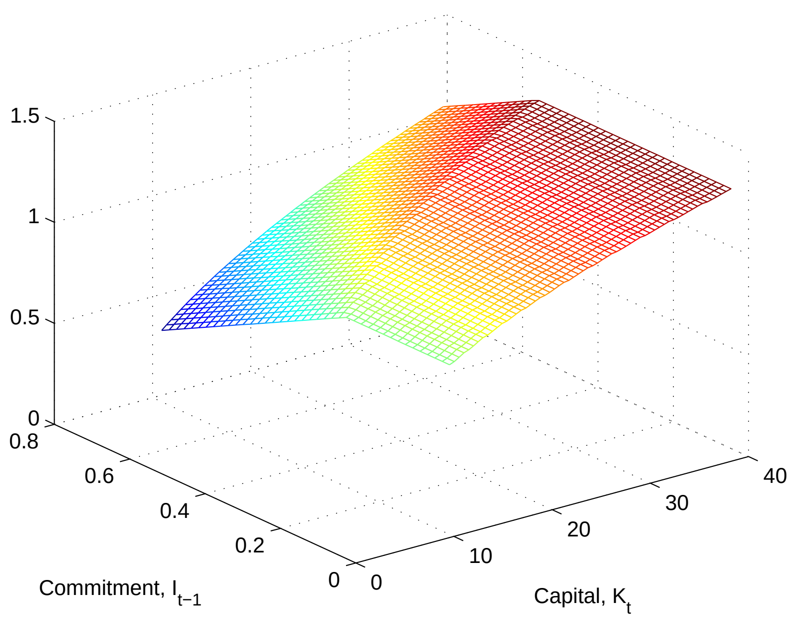

Table 1: EGARCH Volatility Estimates

This table summarizes EGARCH estimates for real non-durable plus service consumption growth (NDS), real personal consumption expenditures growth (PCE), real output growth (GDP), and aggregate stock returns as measured by the value-weighted CRSP index. All data is quarterly and covers the period form 1952 to 2011. The EGARCH model postulates that the log variance follows

$$\ln \sigma_t^2 = \kappa + \rho \ln \sigma_{t-1}^2 + \lambda(|z_{t-1}| - \mathbb{E}|z_{t-1}|) + \theta z_{t-1} \qquad z_t = \epsilon_t/\sigma_t \tag{31}$$

where κ is the average log variance, ρ measures the persistence of the log variance, λ the size of the impact of shocks on volatility and θ its sign. Innovations are standard normal, $\epsilon_t \sim \mathcal{N}(0,1)$.

	NDS	PCE	GDP	Returns
$\bar{g}$ (%)	0.47	0.55	0.46	1.92
	(0.07)	(0.08)	(0.09)	(0.61)
ρ_g	0.44	0.35	0.40	0.17
	(0.08)	(0.07)	(0.07)	(0.08)
κ	-4.04	-1.80	-1.22	-1.52
	(1.94)	(0.96)	(0.67)	(0.58)
ρ	0.63	0.82	0.87	0.70
	(0.18)	(0.10)	(0.07)	(0.11)
λ	0.50	0.35	0.42	0.28
	(0.14)	(0.12)	(0.11)	(0.17)
θ	-0.16	-0.16	-0.19	-0.34
	(0.07)	(0.07)	(0.08)	(0.09)
Mean (%)	0.84	0.83	0.76	2.85
Std. Dev. (%)	0.50	0.74	0.94	8.41
Skewness	-0.27	-0.38	-0.35	-0.56
Kurtosis	4.38	4.92	4.39	3.82

Table 2: Parameters of the Benchmark Calibration

This table summarizes the benchmark calibration of the model. All values are quarterly.

Parameter	Symbol	Value
Preferences:		
Discount rate	β	0.995
Risk aversion	γ	10
EIS	ψ	1.5
Technology:		
Growth rate	g	0.004
Std. deviation	σ	0.03
Production:		
Capital elasticity	α	0.4
Depreciation	δ	0.02
Commitment	w	0.95

Table 3: Model Comparison

This table presents the unconditional moments generated by four versions of the model: Model A and B have no commitment and a low and high EIS, respectively; Model C and D have commitment and and a low and high EIS, respectively. In the table, $\mathbb{E}[R - R^f]$ and $\sigma(R - R^f)$ denote the average excess return and excess return volatility, $\mathbb{E}[R^f]$ and $\sigma(R^f)$ the average risk-free rate and risk-free rate volatility, $\sigma(\Delta \ln C^a)$ the volatility of annual log consumption growth. All moments are annualized. The data column is taken from Bansal and Yaron (2004) and their sample period is 1929-1998.

			Model			
Parameter		Data	A	B	C	D
Risk aversion	γ		10	10	10	10
EIS	ψ		0.5	1.5	0.5	1.5
Project half-life	H (years)		0	0	3.5	3.5
Variable	Statistic					
Excess return	$\mathbb{E}[R - R^f]$ (%)	6.33	0.10	0.23	0.61	3.47
	$\sigma(R - R^f)$ (%)	19.42	0.45	0.78	1.86	9.29
Sharpe-Ratio	$\mathbb{E}[R - R^f]/\sigma(R - R^f)$	0.33	0.23	0.30	0.33	0.37
Risk-free rate	$\mathbb{E}[R^f]$ (%)	0.86	1.56	0.36	2.98	1.20
	$\sigma(R^f)$ (%)	0.97	2.54	1.49	2.09	0.89
Consumtion growth	$\sigma(\Delta \ln C^a)$ (%)	2.93	2.54	2.19	2.09	2.95

Table 4: Cyclicality

This table reports the (unconditional) correlation of realized log consumption growth, $\Delta \ln C_t$, with the conditional volatility of consumption growth, $\sigma_t(\Delta \ln C_{t+1})$, conditional excess returns, $\mathbb{E}_t[R_{t+1} - R^f_{t+1}]$, conditional stock return volatility, $\sigma_t(R_{t+1})$, conditional consumption beta, β_t, and market price of risk, λ_t. Each model is simulated for 5,000 periods.

		Model			
Parameter		A	B	C	D
Risk aversion	γ	10	10	10	10
EIS	ψ	0.5	1.5	0.5	1.5
Project half-life	H (years)	0	0	3.5	3.5
Variable	Statistic				
Cond. consumption volatility	$\sigma_t(\Delta \ln C_{t+1})$	0.09	0.05	-0.15	-0.47
Cond. risk premium	$\mathbb{E}_t[R_{t+1} - R^f_{t+1}]$	0.14	0.39	-0.21	-0.46
Cond. return volatility	$\sigma_t(R_{t+1})$	0.13	0.00	-0.05	-0.47
Cond. consumption beta	β_t	0.13	0.28	-0.07	-0.47
Cond. market price of risk	λ_t	0.13	0.51	-0.14	-0.28

Table 5: Persistence

This table reports the (unconditional) auto-correlation of the price-earnings ratio, P_t/E_t, the book-to-market ratio, K_{t+1}/P_t, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return variance, $\text{Var}_t(R_{t+1})$, conditional consumption beta, β_t, expected consumption growth, $E_t[\Delta \ln C_{t+1}]$, conditional variance of consumption growth, $\text{Var}_t(\Delta \ln C_{t+1})$ of the benchmark model D based on 5,000 periods.

		Lags (quarters)				
Variable	Statistic	1	2	3	4	8
Price-earnings ratio	P_t/E_t	0.97	0.93	0.90	0.87	0.74
Book-to-market ratio	K_{t+1}/P_t	0.92	0.86	0.81	0.77	0.66
Excess return	$\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$	0.44	0.22	0.10	0.04	0.04
Return volatility	$\text{Var}_t(R_{t+1})$	0.48	0.24	0.11	0.06	0.05
Consumption beta	β_t	0.45	0.22	0.09	0.04	0.04
Risk-free rate	R^f_t	0.56	0.39	0.30	0.27	0.22
Consumption growth	$\mathbb{E}_t(\Delta \ln C_{t+1})$	0.51	0.33	0.23	0.19	0.15
	$\text{Var}_t(\Delta \ln C_{t+1})$	0.49	0.25	0.12	0.07	0.06

Table 6: Consumption Volatility

Panel A reports estimates of a GARCH(1,1) process for consumption growth, i.e. the conditional variance of log consumption growth is

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2$$

Panel B reports forecasting regressions of future consumption volatilities on the current log book-to-market ratio and Panel C forecasting regressions of the future log book-to-market ratio on the current consumption volatility. Data: Consumption data is real non-durable plus service expenditures at quarterly frequency for the period 1947-2006. The aggregate book-to-market ratio is from Kenneth French's website. t-statistics are reported in parenthesis and based on Newey-West with 8 lags.

	Data		Model	
Panel A: GARCH(1,1) Estimates				
ω_1	0.17		0.26	
	(15.51)			
ω_2	0.79		0.43	
	(3.47)			
Panel B: Forecasting σ_{t+s}	Slope	R^2	Slope	R^2
$s = 0$	0.28	0.35	0.29	0.83
	(4.65)			
$s = 1$	0.28	0.34	0.18	0.32
	(4.40)			
$s = 2$	0.28	0.35	0.12	0.14
	(4.33)			
$s = 3$	0.28	0.34	0.08	0.06
	(4.07)			
$s = 4$	0.27	0.31	0.05	0.03
	(3.75)			
Panel C: Forecasting $\ln BM_{t+s}$	Slope	R^2	Slope	R^2
$s = 1$	1.29	0.37	0.02	0.27
	(6.98)			
$s = 2$	1.30	0.37	0.01	0.11
	(6.78)			
$s = 3$	1.28	0.36	0.01	0.05
	(6.16)			
$s = 4$	1.25	0.34	0.01	0.02
	(5.81)			

Table 7: Long-Run Predictability

This table presents predictability regression of future excess returns on the log price-earnings ratio, $pe_t = \ln P_t/Y_t$,

$$R^e_{t+s} = a + b\, pe_t + \varepsilon_{t+s}$$

at quarterly frequency. Standard errors are corrected using Newey-West with 8 lags. Data: The price-earnings ratio is from Robert J. Shiller's website. The excess return is the CRSP value-weighted return and the risk-free rate is the 90-day T-Bill. The data is at quarterly frequency spanning the period 1926-2006.

	Data		Model	
s	Slope	R^2	Slope	R^2
1	-4.12	0.02	-6.74	0.01
	(-1.94)			
2	-7.08	0.04	-13.14	0.02
	(-2.30)			
3	-10.12	0.05	-20.73	0.04
	(-2.47)			
4	-15.16	0.08	-28.22	0.06
	(-2.48)			
8	-25.34	0.12	-59.15	0.12
	(-2.59)			
12	-32.55	0.14	-93.12	0.17
	(-2.59)			

Table 8: Conditional First and Second Moments of Returns

This table presents regressions of the conditional first and second moments of stock returns on accounting information. In Panel A, I regress the conditional excess return, $\mathbb{E}_t[R^e_{t+1}]$, on the current book-to-market ratio, $BM_t = K_{t+1}/P_t$, and the current and lagged investment rate, $IR_t = I_t/K_t$ and $IR_{t-1} = I_{t-1}/K_{t-1}$, respectively. Panel B and C contain the same regression specification but the dependent variable is the conditional variance of stock returns, $\text{Var}_t(R_{t+1})^{1/2}$, and future realized excess returns, R_{t+1}, respectively. I simulate 100 times 50 years of data and report cross-simulation averages. All regressions are at quarterly frequency and standard errors are corrected using Newey-West with 8 lags.

	Const.	BM_t	IR_t	IR_{t-1}	R^2
	Panel A: Expected Excess Returns				
1	-0.10	0.15			0.75
	(-20.02)	(20.57)			
2	0.00		-0.01		0.01
	(6.18)		(-0.62)		
3	0.00			0.01	0.01
	(4.93)			(1.03)	
4	0.00		-0.44	0.44	0.18
	(6.16)		(-8.44)	(9.23)	
5	-0.11	0.15	-0.05		0.80
	(-23.85)	(24.42)	(-2.81)		
6	-0.10	0.15	-0.13	0.09	0.80
	(-21.47)	(21.96)	(-4.89)	(4.13)	
	Panel B: Cond. Volatility of Returns				
1	-0.24	0.35			0.69
	(-16.40)	(17.00)			
2	0.01		-0.11		0.03
	(9.96)		(-2.45)		
3	0.01			-0.03	0.01
	(8.63)			(-0.72)	
4	0.01		-1.17	1.10	0.20
	(10.43)		(-8.97)	(9.19)	
5	-0.26	0.37	-0.18		0.78
	(-21.55)	(22.58)	(-4.55)		
6	-0.25	0.36	-0.42	0.24	0.79
	(-19.39)	(20.29)	(-6.04)	(4.30)	

	Const.	BM_t	IR_t	IR_{t-1}	R^2
	Panel C: Realized Excess Returns				
1	-0.25	0.35			0.05
	(-4.36)	(4.39)			
2	0.00		-0.10		0.00
	(3.08)		(-1.48)		
3	0.00			-0.00	0.00
	(1.75)			(-0.01)	
4	0.00		-1.58	1.53	0.03
	(2.35)		(-5.19)	(5.15)	
5	-0.26	0.37	-0.19		0.06
	(-4.30)	(4.33)	(-2.06)		
6	-0.23	0.33	-0.88	0.73	0.07
	(-3.72)	(3.75)	(-3.03)	(2.43)	

Table 9: Short-Run Predictability

This table reports time-series regressions of future realized excess returns (Panel A) and the absolute value of excess returns (Panel B) on the current and lagged investment rate. Data: CRSP value-weighted return, 90-day t-bill rate, real nonresidential investment.

Const.	IR_t	IR_{t-1}	R^2
Forecasting R^e_{t+1}			
0.10	-2.71		0.01
(2.18)	(-1.75)		
0.10	-10.53	7.92	0.02
(2.18)	(-1.53)	(1.21)	
Forecasting $\|R^e_{t+1}\|$			
0.04	0.83		0.00
(1.08)	(0.75)		
0.03	-11.50	12.48	0.05
(0.89)	(-2.55)	(2.60)	